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## **A MATHEMATICAL PROBLEM: HOW DO WE TEACH MATHEMATICS TO LEP ELEMENTARY STUDENTS?**

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Elementary school teachers are facing a new mathematical problem that is causing great concern: how to teach basic mathematical concepts to the increasing numbers of limited English proficient (LEP) students. With the growing number of LEP students nationally, it is important to evaluate the methods teachers are using to teach these students in general, as well as the particular methods used in the process of teaching mathematics.

In attempting to ascertain what particular methods would be effective in mathematics to teach LEP students, the first issue to resolve is, "Does the ability to speak English affect the acquisition of mathematics skills?" Many hold a common belief that the inability to speak English is not so much a barrier as an inconvenience in regard to the learning of mathematics (Kimball, 1990; Kessler, Quinn, & Hayes, 1985). Is this belief substantiated? It seems to depend on the answers to the following two questions:

1. Is rote computation the primary and sufficient element of mathematical literacy?
2. Can a mathematics teacher effectively teach mathematics without speaking, writing, or requiring the reading of text other than through the use of culturally unbiased numerals and mathematical symbols?

If the answers to these questions are yes, then it is essentially true that the inability to speak English is not a significant barrier to the learning of mathematics. On the other hand, if the answers to these questions are no, then a significant barrier to the learning of mathematics could develop.

Considering the recommendations of the National Council of Teachers of Mathematics (NCTM) standards for stressing understanding rather than rote computations (NCTM, 1989) and considering the proven relationship between language skills and mathematics (Aiken, 1972; Cuevas, 1984; Dawe, 1983; Kessler et al., 1985), it seems evident that the belief that inability or limited ability to speak English has a minimal effect on the learning of mathematics is actually a myth. Why is this myth so widely held? The chief justification given is that mathematics is a universal language, and, therefore an individual's knowledge of it is not tied to a particular cultural language. The following brief analysis of the universality of mathematics reveals a few interesting facts.

Secada (1983) and Norman (1988) demonstrated that numerals themselves are not universally the same. Arab or Asian students could easily have trouble translating numerals in their American classroom to meaningful numbers. For example, the numeral 70 will most likely be interpreted by an Arab student as a number other than seventy (see Figure 1).

Even those countries which use Arabic numerals do not necessarily use the same written notation (Secada, 1983). To elaborate on this, 1,539 means one and five hundred thirty-nine one thousandths in the United States. However, in most of Europe and Latin America, 1,539 means one thousand five hundred thirty-nine. Why the large discrepancy? In these particular countries, the period (.) is used in numbers over 999 much as we use the comma(.). The comma is used to indicate decimals much as we use the period (Moeller & Leidloff, 1988).

*Figure 1. English, Arabic, and Chinese/Japanese/Korean Numerals.*

*The contents of this table have been omitted because of the inability to reproduce characters using the ASCII character set.*

Even the simple reading of a numeral comes into play. Recall the numeral 70 which was previously stated to have a strong chance of being interpreted differently by an Arab student than by an American student. Using Figure 1, an American teacher might expect that the confusion would result in the student internalizing the number 65, but in actuality, the student would probably be thinking of the number 56. Why? For the simple reason that Arabic readers, like Chinese readers, read from right to left (Secada, 1983). Obviously, this could cause a student difficulty in an American classroom.

Culture can also interfere in the learning of mathematical concepts in the classroom. One Native American culture does not have a concept for line (Lovett, 1980), and one South American culture does not have a concept for such numbers as 4 and 5. Instead they have conceptualized numbers 1, 2, and many, and the Hmong culture does not have a concept for fractions (Kimball, 1990). These instances demonstrate how culture can interfere with the learning of mathematical concepts.

The National Council of Teachers of Mathematics (1991) acknowledges the potential for language ability to create barriers to learning mathematics and the need for teachers to attend to the role language plays in students' understanding of mathematics. The Council goes on to state, "Teachers' knowledge of their students' cultural backgrounds and the implications of this knowledge for their teaching is crucial in recognizing the impact of language on learning" (p. 146).

Research on children K-3 (Cantieni & Tremblay, 1979) has shown that, "the language skills needed for mathematics were two years ahead of the official system" (p. 247). Or to put it another way, if you were working on mathematical problems in the third grade, you would need a fifth grade reading ability to adequately comprehend the problems.

To understand the relationship of language and mathematics, it is necessary to understand the main components of language as it is used in the mathematics classroom. These components include vocabulary, syntax, semantic properties, and discourse (Dale & Cuevas, 1987). Vocabulary in the mathematics classroom not only includes specialized terms such as quotient, multiplication, divisor, denominator, minuend, and subtraction but also everyday terms that take on new meaning when used in a mathematical context such as rational, even, table, column, product, and quarter (see Figure 2). Added to this are the homophones or words that sound like other commonly used words, such as sum and some, and words that may be hard to distinguish for LEP students, such as addition and audition, angle and ankle, factor and factory (Garbe, 1985).

*Figure 2. Selected Vocabulary for Teaching Mathematics at the Primary Level.*

| ENGLISH          | SPANISH                   |
|------------------|---------------------------|
| To count         | <i>Contar</i>             |
| Number           | <i>Número</i>             |
| Number sentence  | <i>Frase de número</i>    |
| Before           | <i>Antes</i>              |
| After            | <i>Después</i>            |
| To add           | <i>Sumar</i>              |
| Sum              | <i>Suma</i>               |
| Plus             | <i>Más</i>                |
| To subtract      | <i>Restar, sustraer</i>   |
| To take away     | <i>Llevarse</i>           |
| Multiply         | <i>Multiplicar(se)</i>    |
| Divide           | <i>Dividir(se)</i>        |
| Equals           | <i>Iguala</i>             |
| Same as          | <i>Lo mismo que</i>       |
| How many?        | <i>Cuántos?</i>           |
| Steps            | <i>Pasos</i>              |
| Value            | <i>Valor</i>              |
| Place Value      | <i>Valor de lugar</i>     |
| Ones             | <i>Unidades</i>           |
| Tens             | <i>Decenas</i>            |
| In               | <i>En</i>                 |
| More             | <i>Más</i>                |
| Less             | <i>Menos</i>              |
| Exchange         | <i>Cambio</i>             |
| Trade            | <i>Trocar</i>             |
| Group(s)         | <i>Grupo(s)</i>           |
| Shaded           | <i>Matizó</i>             |
| Greater than     | <i>Más grande que</i>     |
| Which is ...?    | <i>Cuál es ...?</i>       |
| Which are ?      | <i>Cuáles son ...?</i>    |
| Which has ...?   | <i>Cuál tiene ...?</i>    |
| Which have ...?  | <i>Cuáles tienen ...?</i> |
| What comes next? | <i>Que viene proximo?</i> |
| Order            | <i>Orden</i>              |
| First            | <i>Primero</i>            |
| Second           | <i>Segundo</i>            |
| Third            | <i>Tercero</i>            |
| Fourth           | <i>Cuarto</i>             |
| Fifth            | <i>Quinto</i>             |
| Sixth            | <i>Sexto</i>              |
| Seventh          | <i>Séptimo</i>            |
| Eighth           | <i>Octavo</i>             |
| Ninth            | <i>Noveno</i>             |
| Tenth            | <i>Décimo</i>             |
| Last             | <i>Último</i>             |

|            |                         |
|------------|-------------------------|
| Minute     | <i>Minuto</i>           |
| Month      | <i>Mes</i>              |
| Week       | <i>Semana</i>           |
| Year       | <i>Año</i>              |
| Measure    | <i>Medir</i>            |
| Remainder  | <i>Resto</i>            |
| Even       | <i>Par</i>              |
| Odd        | <i>Impar, nones</i>     |
| Digit      | <i>Dígito</i>           |
| Times      | <i>Multiplicado por</i> |
| Heavier    | <i>Más pesado</i>       |
| Lighter    | <i>Menos pesado</i>     |
| Bigger     | <i>Más grande</i>       |
| Smaller    | <i>Más pequeño</i>      |
| Longer     | <i>Más largo</i>        |
| Shorter    | <i>Más corto</i>        |
| Give       | <i>Dar</i>              |
| Regrouping | <i>Reagrupando</i>      |
| Circle     | <i>Círculo</i>          |
| Round      | <i>Redondo</i>          |
| Square     | <i>Cuadro</i>           |
| Rectangle  | <i>Rectángulo</i>       |
| Triangle   | <i>Triángulo</i>        |
| Line       | <i>Línea</i>            |
| Decimal    | <i>Decimal</i>          |
| Fraction   | <i>Fracción</i>         |
| Part       | <i>Parte</i>            |
| What       | <i>Que</i>              |
| Row        | <i>Fila</i>             |
| Column     | <i>Columna</i>          |
| Box        | <i>Caja</i>             |

Problems arise at yet another level for the LEP students. These problems are related to syntax, for example, sentence structure and semantic components of language in the mathematics class. They cause difficulties for LEP students in two particular areas. One area relates to the lack of a one-to-one correspondence between mathematical symbols and the words they represent (Kessler et al., 1985). The following two problems illustrate this:

- John had 6 baskets that he divided his 24 apples into equally. How many were in each basket?
- Mary had fifteen apples that she divided into three baskets. How many apples were in each basket?

To solve the first problem, students should write  $6/24$ . Many students, particularly LEP students, will key in on the number words, the arithmetic operation, and their order. Thus, when they attempt to solve the second problem where they should write  $3)15$ , the chances are high that they will write  $15)3$ , using the pattern and the one-to-one correspondence gleaned from the first problem type.

A second area of problems relating to syntax and semantics deals with the use of logical connectors-linking propositions (Kessler et al., 1985), i.e., if ... then, and but, that is, either, even though-which require finely discriminating language skills creating a significant hardship for LEP students. Dawes' (1983) research results showed that the understanding of logical connectors was the one factor that differentiated those students who could successfully reason mathematically from those who could not.

As to the discourse/text component, students are required to combine their linguistic, cognitive, and metacognitive development to successfully comprehend the reading. Thus, in addition to the mathematics skills that they need to solve the problem, students must simultaneously develop reading the comprehension skills in a foreign language while encountering text that is culturally biased. As Garbe (1985) pointed out, students' understanding of mathematics through the reading of their textbooks is limited to a great extent by their vocabulary and reading skills, and this is particularly true of LEP students.

Thus, it becomes obvious that special teaching strategies, methods, or both, should be used when teaching mathematics to LEP students. Teachers must realize that in teaching mathematics to this population of students, they are going to need to incorporate a variety of techniques they have not used in an English speaking classroom. Therefore, the following recommendations are put forth in regard to the teaching of mathematics for meaningful understanding by LEP students. Many of these recommendations fall under the Sheltered English Approach as discussed by Freeman and Freeman (1988). However, the application of their use is not restricted to children with limited English proficiency. They can be applied to a variety of student ability groups as well as to different classroom settings.

1. Stress understanding rather than rote computational procedures.
2. Provide profound exposure to manipulative, concrete, sensory, and hands-on activities, not to replace discussion, but to support it, e.g., integrate a well stocked math lab into the lessons.
3. Use cooperative learning (small group activities) and minimize individual seatwork.
4. Provide opportunities for peer tutoring-preferably by another LEP or bilingual student who understands the concepts. (Caution must be used to discourage copying and to encourage understanding by the LEP students) (Calkins, 1986; Graves, 1983).
5. Include guided practice with close monitoring of students.
6. Use reinforcement, reward, and total motivational systems.
7. Emphasize multicultural referents and relevancy in lessons (Krause, 1983).
8. Use second language texts, materials, and resources as much as possible (Peterson, 1984). (See the attached resource list.)
9. Use limited, simplified instruction (using caution to retain the essence of the original content and problems); limited use of pronouns and adjectives; instruction with pauses and repetition; concerted efforts to be aware of and to explain any culturally-based terms.
10. Use basic mathematics vocabulary in the second language for individualized instruction whenever possible. Note that the vocabulary should not be used to focus on key words but should be used in context to develop understanding.
11. Model expected student behavior.
12. Be aware of how other countries and cultures teach basic mathematical concepts. For example: (a) Hispanic countries most commonly use equal additions method of subtraction. Countries using the French educational system, such as Vietnam, Thailand, and Laos tend to use missing addend method of subtraction, rather than the American regrouping or take away method; (b) Hispanic and Indochinese countries use the alternate algorithm, while in the USA, the traditional long division method is used (Secada, 1983, pp. 22-23).

13. Use programs that are designed to increase hands-on activities and relevancy and minimize abstraction. Programs such as Finding Out/Descubrimiento (Secada & Carey, 1990), Mathematics Their Way (Davison & Schindler, 1988), or the Second Language Approach to Mathematics Skills (Cuevas, 1984) have been developed for LEP students.

14. Use such methods as direct instruction, e.g., Active Mathematics Teaching, or Cognitively Guided Instruction (Secada & Carey, 1990).

While many of these recommendations would be the same whether a student is LEP or an English speaking slow learner, some techniques apply only to LEP students. The success or failure that limited English proficient students encounter in the mathematics classroom will depend to a large extent on their teachers' awareness of the unique problems LEP students bring to the classrooms and the particular teaching techniques most beneficial to these students. This mathematical problem can be solved. The solution will be achieved by teachers who care and who are willing to incorporate into their teaching some of the methods outlined in this paper.

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## **RECOMMENDED TEACHER RESOURCES**

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